

COMMENTS ON THE COMPARISON OF DYNAMIC AND STATIC COMPRESSION DATA*

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Abstract—A detailed discussion is presented of the relationship between dynamic shock wave data at high pressures, acoustic velocity data at low pressures and the static compression data at intermediate pressures reported by Vaidya and Kennedy in the previous paper.

1. INTRODUCTION

IN THE previous paper[1] by Vaidya and Kennedy (VK) it was possible for the first time to make an extensive comparison of the compressions of a number of metals determined by three completely independent experimental methods. The compression data for 16 metals obtained by VK in a static, high-pressure, piston-cylinder apparatus were found to be in agreement with the isothermal compressions calculated from shock wave data up to 45 kbar. Such a comparison is of great importance, for instance, in detecting systematic errors in pressure scales developed in static and dynamic high pressure experiments[2].

The purpose of this note is to identify the shock data used in the comparison by VK and to discuss the validity of extrapolating shock data to low pressures for such a comparison. In connection with this discussion the relationship between measurements of sound velocity at very low pressure and shock velocity data at high pressures is reviewed and a comparison of both kinds of velocity data is made for the metals studied by VK. On the basis of this comparison the consistency between static and dynamic compression data is strengthened.

The reduction of shock compression data on metals to isothermal compression was extensively reviewed in the original survey

article by Rice, McQueen, and Walsh[3]. In this and much of the subsequent work most attention has been given to the reduction of shock data at high pressures and the extrapolation to low pressures was limited to a comparison with the older compressibility data on metals by Bridgman. In a number of these metals low pressure phase transitions are now known and have been detected in shock wave experiments. When no transition is present newer ultrasonic velocity data can now be used in conjunction with high pressure shock data to obtain more accurate compression curves at low pressure. Little thermodynamic equation-of-state information has been extracted directly from shock wave experiments at low pressures in the vicinity of the elastic yield strength because of the more complicated nature of shock wave propagation at low pressures.

2. SHOCK WAVE DATA

Most of the shock compression values quoted by VK are taken from an extensive AIP Handbook tabulation of isothermal compression P - V curves derived from shock data as reported by Keeler[4]. The isotherms were calculated from shock data as fitted in the usual way by a polynomial expansion of the shock velocity U_s in powers of the material or particle velocity U_p ,

$$U_s = C + S U_p + S_2 U_p^2 + \dots$$

The fits used in the handbook tabulation are identified by an asterisk in Table 1 and were

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Table 1. Comparison of shock wave and sonic velocity data

Mat.	Shock						Sonic			
	ρ_x^\dagger (g/cc)	ρ_0 (g/cc)	C (cm/ μ sec)	S	S_2 (μ sec/cm)	S_3 (μ sec/cm) ²	Ref.	C (cm/ μ sec)	S	Ref.
Ag	10.494	10.49	0.322*	1.52			[a]	0.314	1.75	[S1, S2]
Al	2.698	2.78	0.528*	1.50	-0.67	0.41	[b]	0.532	1.48	[S3, S4]
Au	19.302	19.24	0.312*	1.52			[a]	0.300	1.82	[S1, S2]
			0.294	1.87	-1.38		[c]			
Ba	3.65	3.75	0.159	0.88			[d]	0.159		[d]
Ca	1.53	1.56	0.346	0.99			[d]	0.331		[d]
Cd	8.642	8.64	0.238	1.75			[e]	0.237	1.89	[S5, S6]
			0.247*	1.66			[a]			
Cu	8.932	8.90	0.401*	1.47			[a]	0.392	1.62	[S1, S2]
Fe	7.873	7.85	0.460	1.43			[a]	0.460	1.56	[S7, S8]
In	7.286	7.28	0.239*	1.55			[f]	0.239		[S9]
La	6.174	6.135	0.208	1.12			[g]	0.213		[g]
Mo	10.220	10.17	0.514*	1.26			[a]	0.513		[S10, S15]
Ni	8.907	8.86	0.465*	1.45			[h]	0.454		[S11]
Pb	11.341	11.34	0.201*	1.54			[a]	0.199	1.58	[S12, S13]
Sn	7.285	7.29	0.275	1.37			[i]	0.275		[S14]
Ta	16.626	16.67	0.345*	1.20			[a]	0.340		[S15]
Zn	7.134	7.13	0.298*	1.65	-0.26		[a]	0.294		[S16]

[a] VAN THIEL M. and KUSUBOV A. S., *Compendium of Shock Wave Data*, Vol. 1 and 2, UCRL-50108 (1966 plus 1967 supplement). Available from Clearinghouse for Federal Scientific and Technical Information, NBS, U.S. Dept. of Commerce, Springfield, Va 22151.

[b] From unpublished fit to LRL shock data on Al 2024 and sonic data. The data of G. D. Anderson, D. G. Doran, and A. L. Fahrenbruch on a purer alloy Al 1060 is fit by $0.539 + 1.34 U_p$. Below 50 kbar there is a negligible difference in V/V_0 .

[c] PASTINE D. J. and PIACESI D., *J. Phys. Chem. Solids* **27**, 1783 (1966).

[d] LARSON D. B., *Lawrence Radiation Lab., Livermore*, (private communication). With the use of standard flash gap techniques transition pressures of ~ 180 and 70 kbar were established in Ca and Ba respectively. The reason for the high sample densities relative to X-ray values is not known.

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summary fits to U_s-U_p data from several laboratories. The data, their source as well as the summary fits are taken from a Compendium of Shock Wave Data (Table 1, Ref. [a]).

A number of the metals investigated by VK have phase transitions with substantial volume changes below 200 kbar. For five of these, Fe, Sn, Ba, Ca and La, some shock data on the low pressure phase are available. The U_s-U_p fits for these metals are also listed in Table 1.

The correction to the shock compression curve for shock heating was calculated in a standard manner by assuming a Mie-Grüneisen γ which depends on the shape of the zero degree isotherm in PV space. The Dugdale-MacDonald relationship for γ was used because of the average agreement with thermodynamic values of γ at low pressure [3]. However, the thermal pressure corrections which depend on gamma are very small at pressures below 45 kbar. For instance, an extreme case is the isotherm of Pb where the thermal offset between the isotherm and Hugoniot should be large because of the low specific heat and high compressibility of this metal. The Dugdale-MacDonald γ in this case is 30 per cent low. Calculations however show that the compression along the 300°K isotherm is increased by only 2 per cent if gamma is increased by 40 per cent.

Another source of differences between static and shock compression data arises from the different reference density used in each method. The compression calculated from a shock velocity measurement in a sample by the usual conservation relation is strictly speaking relative to the initial bulk density of the sample. In the static compression method the reference density is that of the sample after several preliminary compression cycles which is presumed to be the

X-ray density of the material. Table 1 lists typical initial densities ρ_0 for samples used in shock wave experiments which were used to calculate pressures and compressions. It can be seen that many are several tenths of a per cent lower than the X-ray densities listed in the table. Because the Hugoniot pressure is proportional to ρ_0 , the resulting error in the calculated shock pressure is negligible. The error in compression due to ρ_0 is larger and more difficult to estimate. It is of the order of the deviation from X-ray density divided by the mean density change over the range of shock measurements and is proportional to the Grüneisen γ in the same compression range. Since shock wave data is taken in a range of compressions typically from 15 to 35 per cent, the calculated compression is larger than that of a dense solid by a small amount of the order of 1-2 per cent. Note that this error is similar in magnitude to the random errors in the measurement of shock velocities and is usually ignored.

3. COMPARISON OF SONIC AND SHOCK DATA

The extrapolation of Hugoniot data to low pressures is best evaluated by comparing with ultrasonic data. The ideal $P-V$ Hugoniot for a material of zero strength coincides in slope and curvature with the isentrope (locus of constant entropy) through the initial state. Therefore if material strength effects are very small, the 'bulk' sound speed calculated from the isentropic bulk modulus, B_s , should be equal to C and the linear shock velocity coefficient, S , should be determined by the pressure derivative of B_s along an isentrope,

$$S = \frac{1}{4} \left[\left(\frac{\partial B_s}{\partial P} \right)_s + 1 \right].$$

A large amount of more accurate ultrasonic

*Fits used in Ref. [4].

†TAYLOR A. and KAGLE G. J., X-ray densities taken from *Crystallography Data on Metal and Alloy Structures*, Dover, New York (1963); with the exception of Mo which was calculated from crystal constants in WYKOFF R. W. G., *Crystal Structures*, 2nd Edn. Interscience, New York (1963).

data has become available in recent years and a tabulation of the ultrasonically derived coefficients C and S for metals is also listed in Table 1. $(\partial B_s/\partial P)_s$ is evaluated by means of the thermodynamic relation

$$\left. \frac{\partial B_s}{\partial P} \right|_s = \left. \frac{\partial B_s}{\partial P} \right|_T + \gamma \alpha T \left. \frac{\partial \ln B_s}{\partial \ln V} \right|_P$$

from the measurements of sound velocity changes with both changes in pressure and temperature. An experimental uncertainty of ≤ 1 per cent can be assigned to C judging by agreement between different experimentalists. On the same basis, however, the experimental uncertainty in S is ~ 10 – 20 per cent. The tabulated sonic data on S are from sonic velocity measurements over a pressure range of several kilobars or more.

Among the metals without low pressure phase transitions there is generally a satisfactory agreement between the two dynamic determinations of C , to ≤ 2 per cent. The value determined from shock data on Ag, Au, Cu, and from the summary data fit for Cd exceed the ultrasonic values of C by a somewhat larger amount. In these cases the shock values of S are also lower than usual. Pastine and Piacesi[5] have suggested that this is due to curvature in the Hugoniot at low pressures. They fit shock data on the noble metals to a quadratic U_s-U_p relation holding C fixed at an earlier ultrasonic value. Considerably closer agreement with sonic values of S was found as shown for example in the table for Au. In the case of Cd the shock wave data itself gives evidence of a similar curvature at low pressures. Table 1 contains a fit to lower pressure shock data on Cd which is in good agreement with sonic data. This fit was used for the comparison with static data in the previous paper.

The dynamic compression data on low pressure phases of Ba, Ca and La consists of a small number of Hugoniot points and sound velocity measurements on a polycrystalline sample. The disagreement between the fits

to the shock data and the bulk sound speed in Ca and La are not inconsistent in view of the sparsity of the Hugoniot data. VK find that for Ca the agreement between the shock-derived isothermal compression and static measurements is excellent while in the case of La the disagreement (~ 5 per cent) is the largest of all the metals.

It should be noted here that the shock velocity 'fits' listed in Table 1 for the low pressure phases of Fe and Sn were obtained from measured ultrasonic velocities and shock data at the transition pressure. In the case of Fe, there is extensive but anomalous shock data below 100 kbar on the low pressure phase, which is not adequately understood[6].

Although there is good overall agreement between sonic and extrapolated shock wave velocities at low pressure there is still room for improvement in the agreement by means of material strength corrections. For instance, the above indications of a non-linear U_s-U_p relation in a few metals can also be explained as a strength effect. In addition, although the ultrasonic value of S is not very reliable for some metals, Table 1 indicates that there may be a more widespread difference between sonic and shock wave values of S among the metals. At high pressures ($\geq \frac{1}{2}$ bulk modulus) one may be sure that the strength correction becomes negligible for two reasons. Judging by data on a wide variety of solids at normal densities yield strength should be a fraction of 1 per cent of the bulk modulus. Secondly the yield strength must drop quickly as shock heating brings the metal near the melting temperature. The above agreement between sonic and shock velocities at low pressures suggests that in metals the correction for material strength is small down to much lower pressures. For other types of solids (i.e. ceramics, rocks, etc.) of high yield strength there are discrepancies between sonic and shock data indicating that significant corrections are needed[7]. It is to be hoped that the extensive experimental work being done presently on shock propagation at very low pressures[8]

will clarify the questions about the importance of the strength correction in metals at lower pressures.

4. DISCUSSION AND CONCLUSION

Within the qualifications discussed above it has been shown that the extrapolation shock velocity data for most metals to zero pressure is consistent with sound velocity data to within ~ 2 per cent, comparable with the accuracy of the shock velocity data itself. Therefore the shock compression data is suitable for comparison with the static data of VK in the intermediate pressure range. Except for La the agreement found by VK between the two sets of compression data is everywhere consistent within an experimental error of several tenths of a percent in the static volume measurements and of 2 per cent in the shock compression measurements. It may thus be concluded that all three types of compression data are consistent up to 45 kbar.

Discrepancies between the isothermal compressibility calculated from fits to the static data of VK and from sound velocity data in some metals are apparently not real. In the case of Fe, Rotter and Smith[2] have also noted the differences between the compressibility obtained from sonic velocity data and fits to Bridgman's data. Such differences have usually been attributed to a loss of accuracy in taking differences between volume measurements in the static compression method. These discrepancies may also in part be due to inappropriate methods of fitting or representing static data. In this connection it

should be noted that the simple U_s-U_p expansion used to fit shock wave data over a much larger range of pressures implies a volume dependence of the pressure along the isotherm which is different from either the Bridgman or Murnaghan form used by VK.

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